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For B.Sc. Part I (H)

Book: Theory of Equation

chapter: Character and position of the Roots

Lecture on Determination of Multiple Roots by

Dr. Ram Anek Yadav

Assistant professor, Department of mathematics, Marwari college Darbhanga.

Mobile No — 9708834223

## Determination of Multiple Roots:

We have seen above that multiple root of the order $m$ is a multiple root of order $(m-1)$ of the equation $f'(x)=0$. In order to find such roots we should find the H.C.F of $f(x)$ and $f'(x)$. This H.C.F will give the multiple root of $f(x)$ each repeated $(m-1)$ times. Thus if $(x-2)$ is the H.C.F of $f(x)$ and $f'(x)$. Then $f(x)$ contains $(x-2)^2$ as a factor or 2 is a double root of $f(x)=0$. If $(x-2)^2 \cdot (x-1)^5$ is the H.C.F. of $f(x)$ and $f'(x)$ then $f(x)=0$ has three roots equal to 2 and six roots equal to 1

**Note**: If $f(x)$ and $f'(x)$ have no common factor, then clearly $f(x)=0$ has no equal roots.

## Condition for two or three equal roots:

If $\alpha$ be a double root of $f(x)=0$ then $f(\alpha)=0$ and $f'(\alpha)=0$ and the required condition is obtained by eliminating $\alpha$

between $f(\alpha)=0$ and $f'(\alpha)=0$. Similarly if $\alpha$ is a triple root the required condition can be obtained by eliminating $\alpha$ between $f(\alpha)=0$ & $f'(\alpha)=0$, and $f''(\alpha)=0$.

Example: Solve the equation
$x^5 - 15x^3 + 10x^2 + 60x - 72 = 0$ be testing for equal roots.

Solution: Given that $x^5 - 15x^3 + 10x^2 + 60x - 72 = 0$ ①

$$f(x) = x^5 - 15x^3 + 10x^2 + 60x - 72$$

$$\therefore f'(x) = 5x^4 - 45x^2 + 20x + 60\times 1 - 0 = 0$$

$$\Rightarrow f'(x) = 5[x^4 - 9x^2 + 4x + 12] = 0$$

$$\Rightarrow f'(x) = x^4 - 9x^2 + 4x + 12 = \frac{0}{5} = 0 \quad ②$$

Let us find the H.C.F of $f(x)=0$ and $f'(x)=0$.

| | | | |
|---|---|---|---|
| $x$ | $x^5 - 15x^3 + 10x^2 + 60x - 72$ <br> $x^5 - 9x^3 + 4x^2 + 12x$ | $x^4 - 9x^2 + 4x + 12$ <br> $x^4 - x^3 - 8x^2 + 12x$ | $x$ |
| | $-6x^3 + 6x^2 + 48x - 72$ <br> or $x^3 - x^2 - 8x + 12$ | $x^3 - x^2 - 8x + 12$ <br> $x^3 - x^2 - 8x + 12$ | $1$ |
| | | $\times$ | |

Thus we find the H.C.F of $f(x)=0$ & $f'(x)=0$ is $x^3 - x^2 - 8x + 12 = 0$ or $(x-2)(x^2+x-6)=0$ or $(x-2)^2(x+3)=0$ giving $x = 2, 2, -3$

Now, we know the H.C.F gives a multiple root of $f(x)=0$, $(m-1)$ times. Hence $2$ is triple root and $-3$ is a double root of $f(x)=0$.